Solution 8 :

1°/ $Z = \dfrac{(z_1+z_2)^2}{z_1 z_2} = \dfrac{z_1^2+z_2^2+2z_1z_2}{z_1z_2} = \dfrac{z_1}{z_2}+\dfrac{z_2}{z_1}+2$

or $z_1 = [1,\alpha]$ et $z_2 = [1,\beta]$

d'où $\dfrac{z_1}{z_2} = [1,\alpha-\beta]$ et $\dfrac{z_2}{z_1} = [1,\beta-\alpha]$

$\Rightarrow Z = \cos(\alpha - \beta) + i\sin(\alpha - \beta) + \cos(\beta - \alpha) + i\sin(\beta - \alpha) + 2$

$= 2\cos(\alpha - \beta) + 2$ (car $\cos(-X) = \cos X$ et $\sin(-X) = -\sin X$)

$\Rightarrow Z = 2(\cos(\alpha-\beta)+1) \in \mathbb{R}_+$ car $-1 \leq \cos(\alpha - \beta) \leq 1$.

2°/ $|b|.\overrightarrow{AI} + |a|.\overrightarrow{BI} = \vec{0} \Leftrightarrow |b|(z_1 - z_A) + |a|(z_1 - z_B) = 0$

$\Leftrightarrow z_1 = \dfrac{a|b| + b|a|}{|a|+|b|}$.

Solution 9 :

1°/ $z = \dfrac{1}{2}(\sin\varphi + i2\sin^2\dfrac{\varphi}{2}) = \sin\dfrac{\varphi}{2}(\cos\dfrac{\varphi}{2} + i\sin\dfrac{\varphi}{2})$

$\sin\dfrac{\varphi}{2} = 0 \Leftrightarrow \dfrac{\varphi}{2} = k\pi$ où $k \in \mathbf{Z}$

φ	$-\pi$	0	π
$\sin\dfrac{\varphi}{2}$	$-$	0	$+$

si $\varphi \in [-\pi, 0[$ on a : $z = \left[-\sin\dfrac{\varphi}{2}\ ;\ \dfrac{\varphi}{2}+\pi\right]$

si $\varphi = 0$ on a : $z = 0$

si $\varphi \in]0, \pi]$ on a : $z = \left[\sin\dfrac{\varphi}{2}\ ;\ \dfrac{\varphi}{2}\right]$